

Chapter 4 by N. Kikuchi and J. T. Oden is devoted to classical contact problems in elastostatics. After a brief recall of Signorini's model, the authors introduce simplifications in order to avoid difficulties connected with the existence of a solution. A finite element method is presented and analyzed for these approximate models. The numerical solution, based on a penalty technique, is also discussed. Finally, a computational test (cylindrical punch on a body) is given.

A new friction law is introduced in Chapter 5. It is due to J. T. Oden and E. B. Pires, who are the authors of this last chapter. In order to avoid difficulties connected with existence of a solution in Signorini's model, a nonlocal law—which can be understood as a regularization of Coulomb's law—is suggested. The local value of the normal stress is approximated by “an average” around the concerned point on the boundary. The variational formulation leads to an existence and uniqueness result for this new model. The final section is devoted to a finite element approximation of the model, for which the same numerical test, as in Chapter 4, is checked.

This book appears to be interesting for a reader who is concerned with mathematical aspects of finite element methods applied to some problems arising in mechanics. Furthermore, its presentation is very good and a homogeneity between the different chapters has successfully been obtained by the editors.

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**14[76S05, 76T05, 65N30].**—GUY CHAVENT & JEROME JAFFRE, *Mathematical Models and Finite Elements for Reservoir Simulation—Single Phase, Multiphase and Multicomponent Flows Through Porous Media*, Studies in Mathematics and its Applications, vol. 17, North-Holland, Amsterdam, 1986, xi+376 pp., 23 cm. Price \$80.00/Dfl. 200.00.

The aim of the book is threefold. First, there is a presentation of several mathematical models related to the reservoir simulation problem. Being unable myself to judge the accuracy and the practical relevance of each model, I can nevertheless say that the exposition is simple, very clear, and accessible, even to people with a rather weak physical background. The various models are presented in a synthetic way, using the new feature of the “global pressure”. Clear hints on various practical situations in which one or another model comes into the game gives one the feeling of being “in contact with the real world”.

A second scope of the book is to provide a rigorous mathematical study of some of the simpler problems. In reading this part, I was rather happy to have spent less time in my life on the study of physics and more on the study of functional analysis. The treatment is indeed very well done, clean, precise and reasonably understandable, provided one has some background in functional analysis.

A third aim of the book is to present some of the new finite element techniques in order to deal with some of these problems. The range of methods that are analyzed

is not very wide, but at least the choice falls on methods and techniques which are very recent and effective.

Therefore, in a sense, all three aims of the book have been successfully achieved. Although the field itself is, I think, much wider than what is covered in the 376 pages of this book, I believe that the book can be very useful, both to experts and to beginners in the field.

The contents of the book are organized as follows: Chapter I: Basic laws and models for flow in porous media (Generalities, The geometry of the field, The basic laws for one- and two-phase flow, The basic models, Qualitative behavior of the solution in the no-diffusion and no-capillary pressure case). Chapter II: Slightly compressible monophasic fields (Construction of the pressure equation, Existence and uniqueness theorems, An alternative model of monophasic wells). Chapter III: Incompressible two-phase reservoirs (Introduction, Construction of the state equations, Summary of equations of two-phase flows for incompressible fluids and rock, An alternative model for diphasic wells, Mathematical study of the incompressible two-phase flow problems, The case of fields with different rock types). Chapter IV: Generalization to compressible, three-phase, black oil or compositional models (The two-phase compressible model, The three-phase compressible model, The black oil model, A compositional model). Chapter V: A finite element method for incompressible two-phase flow (Introduction, Approximation of the pressure-velocity equations, Resolution of the algebraic system for pressure-velocity, Approximation of the one-dimensional saturation equation: the case with neither capillary pressure nor gravity, Approximation of the one-dimensional saturation equation in the general case, Approximation of the saturation equation in two dimensions, Notes and remarks).

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**15[65L60, 65M60, 65R20].**—H.-J. REINHARDT, *Analysis of Approximation Methods for Differential and Integral Equations*, Applied Mathematical Sciences, Vol. 57, Springer-Verlag, New York, 1985, xi+390 pp., 23½ cm. Price \$45.00.

This is a major contribution to the literature on the approximate solution of differential and integral equations. Most of the material comes from the research of the author and colleagues during recent years. A unified theory yields general convergence results and error estimates for approximate solutions of linear and nonlinear problems. The theory is applied to finite difference approximations for initial and boundary value problems, projection methods for differential and integral equations, and quadrature methods for integral equations.

The book is divided into four parts: numerical methods and examples, general convergence theory; applications to boundary value problems and integral equations; inverse stability, consistency and convergence for initial value problems.